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ON NONSTATIONARY VIBRATIONS OF PLATES ON AN ELASTIC FOUNDATION

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The problem is solved concerning the vibrations of an infinite elastic, constant-thickness plate covering the boundary of an anisotropic elastic half-space. It is assumed that there is no friction between the plate and the half-space boundary, but constant normal and tangential forces act in the plane of the plate. Nonstationary vibrations are caused by shock loads acting on the plate, which results in the appearance of three kinds of plane shocks in the elastic anisotropic half-space, behind whose fronts the solution is constructed by using ray series [1].

1. We take the direction of the constant forces N_{11}, N_{22} acting in the plane of the plate as the x_1, x_2 axes, respectively, and the normal to the plate as the axis x_3. The system of equations describing the plate vibrations and the dynamic behavior of the anisotropic half-space has the form

$$D\Delta\Delta w + \rho_1 h w^{\cdot\cdot} + N_{\alpha\beta} w_{,\alpha\beta} = q, \quad D = \frac{Eh^3}{12(1-\nu^2)} \tag{1.1}$$

$$\sigma_{ij,j} = \rho v_i^{\cdot\cdot}, \quad \sigma_{ij}^{\cdot} = \lambda_{ijml} v_{m,l} \tag{1.2}$$

Here w is the plate deflection, D is the cylindrical stiffness, h is the thickness, E is the elastic modulus, ν is the Poisson's ratio, ρ_1 is the density of the plate material, $q(x_1, x_2, t)$ is the pressure of the half-space on the plate, σ_{ij} are the stress tensor components, ρ is the density of the half-space material, v_i are components of the displacement velocity vector, λ_{ijml} are isothermal stiffness coefficients of the anisotropic material, the Latin subscripts take on the values $1, 2, 3$ while the Greek subscripts take on $1, 2$ and the points denote the derivative with respect to the time t, the subscript after the comma denotes the derivative with respect to the appropriate coordinate.

At the initial instant, let a velocity dependent on the coordinates x_1, x_2

$$w|_{t=0} = 0, \quad w^{\cdot}|_{t=0} = w_0^{\cdot}(x_\alpha) \tag{1.3}$$

be communicated to points of the plate.

Let us seek the quantity q in the form

$$q = \sum_{k=0}^{\infty} \frac{1}{k!} F_{(k)}(x_\alpha) t^k \tag{1.4}$$

where ($F_{(k)}$ are unknown functions).

A sudden application of pressure to the boundary of the anisotropic half-space results in the generation of plane shocks behind whose fronts the desired functions $Z(x_\alpha, t)$

are represented by power series in $t - x_3 c_{(n)}^{-1} \geqslant 0$, i. e.,

$$Z^{(n)}(x_\alpha, t) = \sum_{k=0}^{\infty} \frac{1}{k!} [Z^{(n)}_{,(k)}] \left(t - \frac{x_3}{c_{(n)}}\right)^k \tag{1.5}$$

Here $[Z^{(n)}_{,(k)}]$ are jumps in the k -th order time derivatives of the functions $Z^{(n)}(x_\alpha, t)$ on the shock fronts, i. e., for $t = x_3 c_{(n)}^{-1}$ the subscript n indicates the ordinal number of the wave.

To determine the coefficients of the ray series (1.5) of the required functions σ_{ij}, v_i, let us differentiate the equations of the system (1.2) k times with respect to time t, and take their difference on different sides of the wave surface. We consequently obtain

$$[\sigma^{(n)}_{ij,\,j(k)}] = \rho\, [v^{(n)}_{i,\,(k+1)}], \quad [\sigma^{(n)}_{ij,\,(k+1)}] = \lambda_{ijml}\, [v^{(n)}_{m,\,l(k)}] \tag{1.6}$$

Taking account of the compatibility condition for the discontinuities of the $(k+1)$ -th order derivatives of some function $Z(x_\alpha, t)$ [2]

$$c_{(n)}\, [Z^{(n)}_{,\,i(k)}] = -[Z^{(n)}_{,\,(k+1)}]\, \nu_i + \frac{\delta\, [Z^{(n)}_{,\,(k)}]}{\delta t}\, \nu_i + [Z^{(n)}_{,\,(k)}]_{,\alpha}\, x_{i,\alpha} \tag{1.7}$$

we have from (1.6) after manipulation

$$\omega^{(n)}_{im}\, [v^{(n)}_{m,\,(k+1)}] = \Omega^{(n)}_{i(k)}, \quad \omega^{(n)}_{im} = \lambda_{ijml} \nu_j \nu_l - \rho c^2_{(n)} \delta_{im} \tag{1.8}$$

$$c_{(n)}\, [\sigma^{(n)}_{ij,\,(k+1)}]\, \nu_j = -\rho c^2_{(n)}\, [v^{(n)}_{i,\,(k+1)}] - \lambda_{ijml} \nu_j \nu_l \frac{\delta [v^{(n)}_{m,\,(k)}]}{\delta t} -$$

$$c_{(n)} \lambda_{ijml} x_{j,\,\alpha} \nu_l\, [v^{(n)}_{m,\,(k)}]_{,\alpha} + F^{(k-1)}_{i(n)}$$

$$\Omega^{(n)}_{i(k)} = 2\lambda_{ijml} \nu_j \nu_l \frac{\delta\, [v^{(n)}_{m,\,(k)}]}{\delta t} + c_{(n)} \lambda_{ijml} (x_{l,\,\alpha} \nu_j + x_{j,\,\alpha} \nu_l)\, [v^{(n)}_{m,\,(k)}]_{,\alpha} - F^{(k-1)}_{i(n)}$$

$$F^{(k-1)}_{i(n)} = \lambda_{ijml} \nu_j \nu_l \frac{\delta^2\, [v^{(n)}_{m,\,(k-1)}]}{\delta t^2} + c_{(n)} \lambda_{ijml} (x_{l,\,\alpha} \nu_j + x_{j,\,\alpha} \nu_l) \times$$

$$\frac{\delta\, [v^{(n)}_{m,\,(k-1)}]_{,\alpha}}{\delta t} + c^2_{(n)} \lambda_{ijml} x_{j,\,\alpha} x_{l,\,\beta}\, [v^{(n)}_{m,\,(k-1)}]_{,\alpha\beta}$$

Here ν_i is the normal to the wave surface, δ_{im} is the Kronecker symbol, $c_{(n)}$ is the velocity of shock propagation, $\delta[Z] / \delta t = \lim\, [([Z]_2 - [Z]_1) / \Delta t]$ as $\Delta t \to 0$, where $[Z]_1$ is the value of the jump in the quantity Z at some point M of the wave surface $\Sigma(t)$, while $[Z]_2$ is the value of the jump at the point of intersection with the surface $\Sigma(t + \Delta t)$ of a vector normal to the surface $\Sigma(t)$ at the point M [2].

We obtain from (1.8) for a jump of zero order

$$\omega^{(n)}_{im}\, [v^{(n)}_m] = 0, \quad c_{(n)}\, [\sigma^{(n)}_{ij}]\, \nu_j = -\rho c^2_{(n)}\, [v^{(n)}_i]$$

It hence follows that the quantities $\rho c^2_{(n)}$ are principal values, while the vectors $[v^{(n)}_m]$ are corresponding principal directions of a symmetric tensor of the second rank $\lambda_{ijml}\, \nu_j \nu_l$.

Taking into account that

$$\lambda_{ijml}\nu_j\nu_l = \sum_{j=1}^{3} \rho c_{(f)}^2 l_i^{(f)} l_m^{(f)}, \quad \omega_{im}^{(n)} l_i^{(n)} = 0, \quad \omega_{im}^{(n)} l_i^{(f)} = \rho\,(c_{(f)}^2 - c_{(n)}^2)\, l_m^{(f)} \quad (f \neq n)$$

where $l_i^{(n)}$ are unit vectors of the principal directions, we find from (1.8)

$$2\rho c_{(n)}^2 \frac{\delta v_{(k)}^{(n,n)}}{\delta t} + c_{(n)} b_\alpha^{(n,n)} v_{(k),\alpha}^{(n,n)} = F_{i(n)}^{(k-1)} l_i^{(n)} - c_{(n)} \sum_{\substack{f=1 \\ (f\neq n)}}^{3} b_\alpha^{(n,f)} v_{(k),\alpha}^{(n,f)} \tag{1.9}$$

$$v_{(k)}^{(n,f)} \rho\,(c_{(f)}^2 - c_{(n)}^2) = 2\rho c_{(f)}^2 \frac{\delta v_{(k-1)}^{(n,f)}}{\delta t} + c_{(n)} \sum_{g=1}^{3} b_\alpha^{(f,g)} v_{(k-1),\alpha}^{(n,g)} - F_{i(n)}^{(k-2)} l_i^{(f)}$$

$$(f \neq n)$$

$$F_{i(n)}^{(k)} l_i^{(f)} = \rho c_{(f)}^2 \frac{\delta^2 v_{(k)}^{(n,f)}}{\delta t^2} + c_{(n)} \sum_{g=1}^{3} b_\alpha^{(g,f)} \frac{\delta v_{(k),\alpha}^{(n,g)}}{\delta t} + c_{(n)}^2 \sum_{g=1}^{3} a_{\alpha\beta}^{(g,f)} v_{(k),\alpha\beta}^{(n,g)}$$

$$v_{(k)}^{(n,f)} = [v_{m,(k)}^{(n)}]\, l_m^{(f)}, \quad b_\alpha^{(n,f)} = \lambda_{ijml} l_i^{(n)} l_m^{(f)} (x_{l,\alpha}\nu_j + x_{j,\alpha}\nu_l)$$

$$a_{\alpha\beta}^{(n,f)} = \lambda_{ijml} l_i^{(f)} l_m^{(n)} x_{j,\alpha} x_{l,\beta}$$

Limiting ourselves henceforth to three terms of the ray series for the desired functions, we obtain from (1.9) for $k = 0, 1, 2$:

$$v_{(0)}^{(n,n)} = f_{(n)}(y_\alpha), \quad v_{(0)}^{(n,f)} = 0 \quad (n \neq f), \quad v_{(1)}^{(n,n)} = g_{(n)}(y_\alpha) + A_{\alpha\beta}^{(n)} f_{(n),\alpha\beta}\, t, \quad v_{(1)}^{(n,f)} = \frac{c_{(n)} b_\alpha^{(n,f)}}{\rho\,(c_{(f)}^2 - c_{(n)}^2)} f_{(n),\alpha} \tag{1.10}$$

$$v_{(2)}^{(n,n)} = k_{(n)}(y_\alpha) + (A_{\alpha\beta}^{(n)} g_{(n),\alpha\beta} + \Gamma_{\alpha\beta\gamma}^{(n)} f_{(n),\alpha\beta\gamma})\, t + A_{\alpha\beta}^{(n)} A_{\gamma\sigma}^{(n)} \times f_{(n),\alpha\beta\gamma\sigma}\, {}^1/_2\, t^2$$

$$v_{(2)}^{(n,f)} = B_{\alpha\beta}^{(n,f)} f_{(n),\alpha\beta} + \frac{c_{(n)} A_{\alpha\beta}^{(n)} b_\gamma^{(n,f)}}{\rho\,(c_{(f)}^2 - c_{(n)}^2)} f_{(n),\alpha\beta\gamma}\, t + \frac{c_{(n)} b_\alpha^{(n,f)}}{\rho\,(c_{(f)}^2 - c_{(n)}^2)} g_{(n),\alpha}$$

$$(n \neq f)$$

$$A_{\alpha\beta}^{(n)} = \frac{1}{2\rho c_{(n)}^2}\left(c_{(n)}^2 a_{\alpha\beta}^{(n,n)} - \frac{b_\alpha^{(n,n)} b_\beta^{(n,n)}}{4\rho}\right) - \frac{1}{2\rho^2} \sum_{\substack{f=1 \\ (f\neq n)}}^{3} \frac{b_\alpha^{(n,f)} b_\beta^{(n,f)}}{c_{(f)}^2 - c_{(n)}^2}$$

$$\Gamma_{\alpha\beta\gamma}^{(n)} = \frac{1}{2\rho c_{(n)}} \sum_{\substack{f=1 \\ (f\neq n)}}^{3} \left\{ \frac{1}{\rho}\left(c_{(n)}^2 a_{\beta\gamma}^{(n,f)} - \frac{1}{2\rho} b_\beta^{(n,n)} b_\gamma^{(n,f)}\right) \frac{b_\alpha^{(n,f)}}{c_{(f)}^2 - c_{(n)}^2} - B_{\alpha\beta}^{(n,f)} b_\gamma^{(n,f)} \right\}$$

$$B_{\alpha\beta}^{(n,f)} = -\frac{1}{\rho\,(c_{(f)}^2 - c_{(n)}^2)} \left\{ c_{(n)}^2 a_{\alpha\beta}^{(n,f)} - \frac{b_\alpha^{(n,f)} b_\beta^{(n,n)}}{\rho} \left(\frac{1}{2} - \frac{c_{(f)}^2}{c_{(f)}^2 - c_{(n)}^2} \right) \right\} + \frac{c_{(n)}^2}{\rho^2 (c_{(f)}^2 - c_{(n)}^2)} \sum_{\substack{g=1 \\ (g\neq n)}}^{3} \frac{b_\beta^{(n,g)} b_\alpha^{(f,g)}}{c_{(g)}^2 - c_{(n)}^2} \quad (n \neq f)$$

Here $f_{(n)}$, $g_{(n)}$, $k_{(n)}$ are derivatives of functions of two arguments $y_1 = x_1 - b_1^{(n,n)} (2\rho c_{(n)})^{-1} t$, $y_2 = x_2 - b_2^{(n,n)} (2\rho c_{(n)})^{-1} t$.

Knowing the quantities $v_{(k)}^{(n,f)}$ we determine

$$[v_{m,(k)}^{(n)}] = \sum_{f=1}^{3} v_{(k)}^{(n,f)} l_m^{(f)}$$

then, for k equal to $k-1$, we find $[\sigma_{ij,(k)}^{(n)}] \nu_j$ from (1.8). Moreover, by using the series (1.5), we compute the components of the displacement vectors u_i and the forces $\sigma_{ij}\nu_j$ behind each shock front

$$u_i^{(n)} = \sum_{k=1}^{3} \frac{1}{k!} [v_{i,(k-1)}^{(n)}] \left(t - \frac{x_3}{c_{(n)}}\right)^k \tag{1.11}$$

$$\sigma_{ij}^{(n)}\nu_j = \sum_{k=0}^{2} \frac{1}{k!} [\sigma_{ij,(k)}^{(n)}] \nu_j \left(t - \frac{x_3}{c_{(n)}}\right)^k$$

and summing these series with respect to n from 1 to 3, we obtain

$$u_i = \sum_{n=1}^{3} u_i^{(n)}, \quad \sigma_{ij}\nu_j = \sum_{n=1}^{3} \sigma_{ij}^{(n)}\nu_j \tag{1.12}$$

The arbitrary functions $f_{(n)}$, $g_{(n)}$, $k_{(n)}$ in (1.12) are determined from the condition that $\sigma_{\alpha j}\nu_j = 0$, $\sigma_{3j}\nu_j = q$ for $x_3 = 0$. Hence, taking account of (1.4), (1.8), (1.10) – (1.12), we find

$$f_{(n)}(x_\alpha) = -\beta_{(n)} F_{(0)}, \quad g_{(n)}(x_\alpha) = -\beta_{(n)} F_{(1)} - \gamma_\alpha^{(n)} F_{(0),\alpha} \tag{1.13}$$

$$k_{(n)}(x_\alpha) = -\beta_{(n)} F_{(2)} - \gamma_\alpha^{(n)} F_{(1),\alpha} - \varkappa_{\alpha\beta}^{(n)} F_{(0),\alpha\beta}$$

Here

$$\beta_{(n)} = \frac{\Delta_{3n}}{\rho c_{(n)} \delta}, \quad \gamma_\alpha^{(n)} = \frac{1}{\rho^2 c_{(n)} \delta^2} \sum_{l=1}^{3} M_{i\alpha}^{(l)} \Delta_{in} \frac{\Delta_{3l}}{c_{(l)}}$$

$$\varkappa_{\alpha\beta}^{(n)} = \frac{\Delta_{in}}{\rho c_{(n)} \delta} \sum_{l=1}^{3} \frac{1}{\rho c_{(l)} \delta} \left(M_{i\alpha}^{(l)} \sum_{m=1}^{3} M_{j\beta}^{(m)} \Delta_{jl} \frac{\Delta_{3m}}{\rho c_{(m)} \delta} + G_{i\alpha\beta}^{(l)} \Delta_{3l} \right)$$

$$M_{i\alpha}^{(n)} = \frac{1}{2} b_\alpha^{(n,n)} l_i^{(n)} - \lambda_{ijml} \nu_l x_{j,\alpha} l_{(m)}^{(n)} - c_{(n)}^2 \sum_{\substack{f=1 \\ (f \neq n)}}^{3} \frac{b_\alpha^{(n,f)}}{c_{(f)}^2 - c_{(n)}^2} l_i^{(f)}$$

$$G_{i\alpha\beta}^{(n)} = -\rho c_{(n)} A_{\alpha\beta}^{(n)} l_i^{(n)} + \frac{1}{4\rho c_{(n)}} b_\alpha^{(n,n)} b_\beta^{(n,n)} l_i^{(n)} - \frac{1}{2\rho c_{(n)}} \lambda_{ijml} (\nu_j x_{l,\beta} +$$

$$\nu_l \nu_{j,\beta}) b_\alpha^{(n,n)} l_m^{(n)} + c_{(n)} \lambda_{ijml} x_{j,\alpha} x_{l,\beta} l_m^{(n)} + \sum_{\substack{f=1 \\ (f \neq n)}}^{3} \left\{ -\rho c_{(n)} B_{\alpha\beta}^{(n,f)} l_i^{(f)} + \right.$$

$$\left. \frac{1}{\rho (c_{(f)}^2 - c_{(n)}^2)} \left(\frac{b_\alpha^{(n,f)} b_\beta^{(n,n)} c_{(f)}^2}{2 c_{(n)}} l_i^{(f)} - c_{(n)} \lambda_{ijml} \nu_l x_{j,\beta} b_\alpha^{(n,f)} l_m^{(f)} \right) \right\}$$

Δ_{ij} is the cofactor to the elements of the matrix $\delta = \| l_i^{(j)} \|$.

By using (1.13) we obtain the expression for u_i in terms of $F_{(0)}$, $F_{(1)}$, $F_{(2)}$.

The unknown functions $F_{(0)}$, $F_{(1)}$, $F_{(2)}$ are found from the condition of continuity of the normal displacements of the plate and anisotropic half-space for $x_3 = 0$. From this condition we have

$$w = -\lambda_3 F_{(0)} t - (\lambda_3 F_{(1)} + \mu_{3\alpha} F_{(0),\alpha}) \frac{t^2}{2} - (\lambda_3 F_{(2)} + \mu_{3\alpha} F_{(1),\alpha} + \nu_{3\alpha\beta} F_{(0),\alpha\beta}) \frac{t^3}{6} \tag{1.14}$$

$$\lambda_m = \sum_{n=1}^{3} \beta_{(n)} l_m^{(n)}, \quad \mu_{m\alpha} = \frac{1}{\rho} \sum_{\substack{n=1 \\ (n\neq f)}}^{3} \sum_{f=1}^{3} \frac{c_{(n)} l_m^{(f)} b_\alpha^{(n,f)}}{c_{(f)}^2 - c_{(n)}^2} \beta_{(n)} + \sum_{n=1}^{3} \gamma_\alpha^{(n)} l_m^{(n)}$$

$$\nu_{m\alpha\beta} = \frac{1}{\rho} \sum_{\substack{n=1 \\ (n\neq f)}}^{3} \sum_{f=1}^{3} \left(\frac{c_{(n)} l_m^{(f)} b_\alpha^{(n,f)}}{c_{(f)}^2 - c_{(n)}^2} \gamma_\beta^{(n)} + B_{\alpha\beta}^{(n,f)} l_m^{(f)} \beta_{(n)} \right) + \sum_{n=1}^{3} \varkappa_{\alpha\beta}^{(n)} l_m^{(n)}$$

Since w is the plate deflection, then (1.14) can satisfy the initial conditions (1.3) and the plate vibrations equation (1.1). The initial conditions are satisfied if we set $F_{(0)} = -w_0^{\cdot} \lambda_3^{-1}$. In order to satisfy the vibrations equation, (1.14) must be substituted into (1.1) and terms with identical powers of t must be equated. We consequently obtain

$$F_{(1)} = \frac{w_0^{\cdot}}{\lambda_3^2 \rho_1 h} + \frac{\mu_{3\alpha}}{\lambda_3^2} \dot{w}_{0,\alpha} \tag{1.15}$$

$$\rho_1 h \lambda_3 F_{(2)} = \left(N_{\alpha\beta} - \frac{\rho_1 h}{\lambda_3^2} \mu_{3\alpha} \mu_{3\beta} + \frac{\rho_1 h}{\lambda_3} \nu_{3\alpha\beta} \right) \dot{w}_{0,\alpha\beta} - 2 \frac{\mu_{3\alpha}}{\lambda_3^2} \dot{w}_{0,\alpha} - \frac{w_0^{\cdot}}{\lambda_3^2 \rho_1 h} + D \Delta\Delta w_0^{\cdot}$$

Knowing $F_{(0)}$, $F_{(1)}$, $F_{(2)}$ we find the required deflection w by means of (1.14).

2. Let us assume that the half-space material is a hexagonal zinc crystal [3]. In this case, two principal values of the symmetric tensor $\lambda_{ijml} \nu_j \nu_l$ coincide $(\rho c_{(1)}^2 = \rho c_{(2)}^2 = \lambda_{1313})$, and their corresponding principal values lie in the plane $x_1 x_2$ (the principal direction corresponding to the third principal value $\rho c_{(3)}^2 = \lambda_{3333}$ coincide with the axis x_3).

Considering the x_1, x_2, x_3 axes to be the principal axes, we find

$$l_i^{(i)} = 1, \quad l_i^{(j)} = 0 \quad (i \neq j), \quad b_\alpha^{(\gamma,\sigma)} = b_\alpha^{(3,3)} = 0 \tag{2.1}$$

$$b_\alpha^{(\beta,3)} = b_\alpha^{(3,\beta)} = l_\alpha^{(\beta)} (\lambda_{1313} + \lambda_{1133}), \quad a_{\alpha\alpha}^{(i,i)} = \lambda_{\alpha i \alpha i}$$

$$a_{12}^{(1,2)} = a_{21}^{(2,1)} = \lambda_{1212}, \quad a_{21}^{(1,2)} = a_{12}^{(2,1)} = \lambda_{1122}$$

$$a_{\alpha\alpha}^{(1,2)} = a_{\alpha\alpha}^{(2,1)} = a_{\alpha\beta}^{(\gamma,3)} = a_{\beta\alpha}^{(3,\gamma)} = a_{12}^{(i,i)} = a_{21}^{(i,i)} = 0$$

Taking account of the relationships (2.1), we obtain from (1.9)

$$v_{(0)}^{(3,3)} = f_{(3)}(\varkappa_\alpha), \quad v_{(0)}^{(3,\beta)} = v_{(0)}^{(1,3)} = 0, \quad v_{(0)}^{(1,\beta)} = f_{(\beta)}(x_\alpha) \tag{2.2}$$

$$v_{(1)}^{(3,3)} = g_{(3)}(\varkappa_\alpha) + \frac{a}{2\rho} f_{(3),\alpha\alpha} t, \quad v_{(1)}^{(3,\beta)} = -c_{(3)} b f_{(3),\beta}$$

$$v_{(1)}^{(1,3)} = c_{(1)} b f_{(\alpha),\alpha}, \quad v_{(1)}^{(1,\beta)} = g_\beta(x_\alpha) + \frac{1}{2\rho} (d f_{(\alpha),\beta\alpha} + \lambda_{1212} f_{(\beta),\alpha\alpha}) t$$

$$v_{(2)}^{(3,3)} = k_{(3)}(\varkappa_\alpha) + \frac{a}{2\rho} g_{(3),\alpha\alpha} t + \frac{a^2}{8\rho^2} f_{(3),\alpha\alpha\beta\beta} t^2$$

$$v_{(2)}^{(3,\beta)} = -bc_{(3)}g_{(3),\beta} - c_{(3)}\frac{ba}{2\rho}f_{(3),\alpha\alpha\beta}t$$

$$v_{(2)}^{(1,3)} = c_{(1)}bg_{(\alpha),\alpha} + \frac{c_{(1)}b}{2\rho}(d + \lambda_{1212})f_{(\alpha),\alpha\beta\beta}t$$

$$v_{(2)}^{(1,\beta)} = k_{(\beta)}(x_\alpha) + \frac{1}{2\rho}(dg_{(\alpha),\alpha\beta} + \lambda_{1212}g_{(\beta),\alpha\alpha})t +$$
$$\frac{1}{8\rho^2}\{d(d + 2\lambda_{1212})f_{(\alpha),\alpha\beta\sigma\sigma} + \lambda_{1212}^2 f_{(\beta),\alpha\alpha\sigma\sigma}\}t^2$$

$$a = be + \lambda_{1313}, \quad b = (\lambda_{3333} - \lambda_{1313})^{-1}e$$

$$d = \lambda_{1212} + \lambda_{1122} - be, \quad e = \lambda_{1313} + \lambda_{1133}$$

$$v_{(k)}^{(3,m)} = [v_{m,(k)}^{(3)}], \quad v_{(k)}^{(1,m)} = [v_{m,(k)}^{(1)}]$$

The subscripts 3, 1 in the last two relationships correspond to waves being propagated at the velocities $c_{(3)}$, $c_{(1)}$.

The arbitrary functions $f_{(n)}$, $g_{(n)}$, $k_{(n)}$ are defined thus

$$f_{(3)} = -\frac{F_{(0)}}{\rho c_{(3)}}, \quad f_{(\beta)} = 0, \quad g_{(3)} = -\frac{F_{(1)}}{\rho c_{(3)}} \tag{2.3}$$

$$g_{(\beta)} = -\frac{c_{(1)}(1+b)}{\rho c_{(3)}}F_{(0),\beta}$$

$$k_{(3)} = -\frac{1}{\rho c_{(3)}}\left\{F_{(2)} + \frac{F_{(0),\alpha\alpha}}{\rho c_{(3)}}\left[c_{(3)}\left(\frac{a}{2} - \lambda_{1133}b\right) + c_{(1)}(1+b)(\lambda_{1133} - \lambda_{3333}b)\right]\right\}$$

$$k_{(\beta)} = -\frac{c_{(1)}(1+b)}{\rho c_{(3)}}F_{(1),\beta}$$

By using (2.2), (2.3), (1.1), we write the plate deflection in the form

$$w = -\frac{F_{(0)}}{\rho c_{(3)}}t - \frac{1}{2}\frac{F_{(1)}}{\rho c_{(3)}}t^2 - \frac{1}{6\rho c_{(3)}}(F_{(2)} + \varepsilon F_{(0),\alpha\alpha})t^3 \tag{2.4}$$

$$\varepsilon = \frac{1}{\rho c_{(3)}}\left[c_{(3)}\left(\frac{a}{2} - \lambda_{1133}b\right) + c_{(1)}(1+b)\lambda_{1133} - \right.$$
$$\left. c_{(1)}c_{(3)}\rho b(1+b)(c_{(3)} - c_{(1)})\right]$$

$$F_{(0)} = -\rho c_{(3)}w_0^{\cdot}, \quad F_{(1)} = \frac{\rho^2 c_{(3)}^2}{\rho_1 h}w_0^{\cdot}, \quad F_{(2)} = -\frac{\rho^3 c_{(3)}^3}{\rho_1^2 h^2}w_0^{\cdot} +$$
$$\frac{\rho c_{(3)}}{\rho_1 h}D\Delta\Delta w_0^{\cdot} + \rho c_{(3)}\varepsilon w_{0,\alpha\alpha}^{\cdot} + \frac{\rho c_{(3)}}{\rho_1 h}N_{\alpha\beta}w_{0,\alpha\beta}^{\cdot}$$

If

$$w_0^{\cdot} = v_0 \sin l_1x_1 \sin l_2x_2 \; (v_0,\ l_1,\ l_2 = \text{const}), \qquad N_{12} = 0$$

then

$$w = v_0\left[t - \frac{1}{2}\frac{\rho c_{(3)}}{\rho_1 h}t^2 + \frac{1}{6\rho_1 h}\left(\frac{\rho^2 c_{(3)}^2}{\rho_1 h} + r\right)t^3\right]\sin l_1x_1 \sin l_2x_2 \tag{2.5}$$

$$r = -D(l_1^2 + l_2^2) + (N_{11}l_1^2 + N_{22}l_2^2)$$

The dependence of the dimensionless deflection $w^* = wc_{(3)}(v_0h)^{-1}$ on the dimensionless time $t^* = tc_{(3)}h^{-1}$ is represented in Fig. 1. Curves 1 — 4 correspond to the following values of $r^* = rh\rho_1^{-1}c_{(3)}^{-2}$: 11.19, 2.19, —0.81, —3.81.

The ratio $\rho\rho_1^{-1}$ was assumed to equal 0.9.

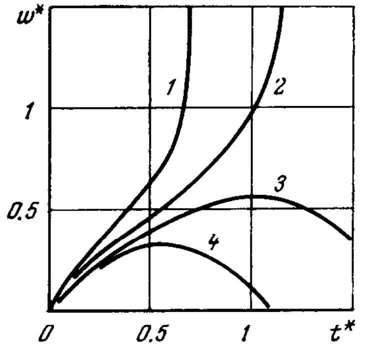

Fig. 1

It is seen that the plate deflection depends on the sign of the quantity r^*: for positive r^* (this corresponds to an excess of the normal forces over the Euler critical value [4]) it grows monotonically with time, and for negative r^* it passes through a maximum.

Therefore, if $w_0^*(x_\alpha)$ is a continuous function differentiable an infinite number of times in the whole x_1x_2 plane, then the problem of plate vibrations can be solved completely by using ray series.

If the initial velocity (initial stress) applied to the plate has one or more lines of discontinuity on which the initial velocity itself ot its derivatives of any order vary by a jump, then the method elucidated is inapplicable. This is associated with the displacement of lines of discontinuity both along the plate (at an infinite velocity) and along the half-space boundary (at the Rayleigh wave velocity), which is not taken into account by the solution obtained.

REFERENCES

1. Achenbach, J.D. and Reddy, D.P., Note on wave propagation in linearly viscoelastic media. Z. angew. Math. und Phys., Vol. 18, No. 1, 1967.

2. Thomas, T., Plastic Flow and Fracture in Solids. " Mir", Moscow, 1964.

3. Nye, J., Physical Properties of Crystals and their Description by Using Tensors and Matrices. "Mir", Moscow, 1967.

4. Bolotin, V.V., Novichkov, Iu.N. and Shvaiko, Iu. Iu., Theory of Aerohydroelasticity. In: Strength. Stability. Vibrations. Vol. 3, Mashinostroenie, Moscow, 1968.

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